TECHNICAL REVIEW

No. 3 — 1981

Contents

CEPSTRUM ANALYSIS

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ABSTRACT

The cepstrum exists in various forms but all can be considered as a spectrum of a logarithmic (amplitude) spectrum. This means that it can be used for detection of any periodic structure in the spectrum, e.g. from harmonics, sidebands, or the effects of echoes. It is also shown, however, that effects which are convolved in the time signal (multiplied in the spectrum) become additive in the cepstrum, and subtraction there results in a deconvolution.

After a discussion of the basic theory, this paper describes many of the applications of the cepstrum, including the study of signals containing echoes (land-based and marine seismology, aero engine noise, loudspeaker measurements) speech analysis (formant and pitch tracking, vocoding) and machine diagnostics (detection of harmonics and sidebands).

The appendices describe how the calculations can be carried out using the B & K FFT analyzers together with a desktop calculator.

SOMMAIRE

Le cepstre existe sous différentes formes, mais toutes peuvent être considérées en réalité comme le spectre d'un spectre logarithmique (en amplitude). Cela veut dire qu'il peut servir à la détection de toute structure périodique dans le spectre, comme, par exemple, des harmoniques, des bandes latérales ou l'influence d'échos. On montre également, cependant, que les effets qui sont soumis à une convolution dans le signal temporel (multiplication dans le spectre) deviennent additifs dans le cepstre et que la soustraction dans ce dernier résulte en une déconvolution.

Après une discussion de la théorie de base, on trouvera dans cet article de nombreuses applications du cepstre, notamment l'étude de signaux contenant des échos (sismologie terrestre et marine, bruits de moteur d'avion, mesures sur les haut-parleurs) , l'analyse phonique (poursuite du "formant" ou du "pitch" de la voix, codage vocal) et les diagnostics de machines (détection des harmoniques et des bandes latérales).

On trouvera dans les appendices la description de la façon dont les calculs peuvent être effectués à l'aide des analyseurs FFT B & K et d'une calculatrice de bureau.

ZUSAMMENFASSUNG

Es existieren verschiedene Formen der Cepstrum-Analyse, alle können jedoch als das Spektrum des logarithmischen (Amplituden-)Spektrums betrachtet werden. D.h. sie lassen sich dazu benutzen, jegliche Art einer Periodizität im Spektrum, wie z.B. Harmonische, Seitenbänder oder Effekte durch Echos, zu erkennen. Es wird ebenso gezeigt, daß eine Faltung im Zeitsignal (Multiplikation im Spektrum) im Cepstrum zu einer Addition führt und daß durch Subtraktion des Effekts das Zeitsignal entfaltet werden kann.
Nach Diskusion der grundlegenden Theorie werden eine Reihe von Anwendungen der Cepstrumanalyse (u.a. Untersuchung von Signalen, die Echos enthalten (seismographische Messungen auf dem Festland und der See, Fluglärm, Messungen an Lautsprechern), Sprachanalyse (Folgen von Formant und Tonhöhe) und Fehlerdiagnose an Maschinen (Auffinden von Harmonischen und Seitenbändern) beschrieben.
Im Anhang wird beschrieben, wie B & K-FFT-Analysatoren in Verbindung mit einem Tischrechner für die Durchführung der nötigen Berechnungen eingesetzt werden können.

Introduction

The cepstrum was first proposed as far back as 1963 (Ref. [1]) and was at that time defined as "the power spectrum of the logarithmic power spectrum." The intended application at that time was to seismic signals because it was realised that it would give information about echoes and that this in turn would help to determine the depth of the hypocentre of a seismic event.

The reason for defining the cepstrum as above is not entirely clear, as even in the original paper it is compared with the autocorrelation function, which can be obtained as the inverse Fourier transform of the power spectrum. Later, another definition of the cepstrum was given as "the inverse Fourier transform of the logarithmic power spectrum", thus making its connection with the autocorrelation clearer. At about the same time, another cepstrum-like function was defined as the "inverse Fourier transform of the complex logarithm of the complex spectrum"(Ref. [2]) and to distinguish it from the above cepstra it was called the "complex cepstrum", while they were renamed "power cepstra". Ref. [3] contains a very good discussion of the definitions and properties of the various forms of the cepstrum, and a guide to some of the applications. The aim of this paper is to summarize that material, adding a couple of newer applications, and also indicating how the cepstrum calculations can be performed using a modern FFT-analyzer in conjunction with a desk-top calculator. This represents a relatively low-cost system which is as powerful and fast as a mini-computer based system, but which at the same time is more flexible in its uses and gives more direct contact with the signals being analyzed. Details of the calculation procedures are given in Appendices A and B.

The applications of the cepstrum can be divided into the purely diagnostic, such as the determination of an echo delay time, or sideband spacing, from the position of a peak in the cepstrum, and those involving editing, where by removal of certain components in the cepstrum it is possible to remove information about their causes. This would for example include removal of the effects of echoes from a spectrum, or time signal, or in speech analysis removal of voice effects leaving only the resonances of the vocal tract, the so-called "formants".

For the diagnostic applications, either definition of the power cepstrum may be used, whereas for the applications involving editing it is essential to use the definitions based on the inverse Fourier transform (which can be followed by a forward transform after editing). Where it is desired to return to the time function (or include phase information in the frequency spectrum) it is necessary to use the complex cepstrum.

The applications discussed here include the processing of signals containing echoes (seismic and underwater signals, measurements on loudspeakers in a reverberant environment, aero engine noise including ground reflections, measurement of the properties of a reflecting surface) speech analysis (formant and voice pitch tracking, vocoding and speech synthesis) and machine diagnosis (determination and monitoring of families of harmonics and sidebands, for example in gearbox and turbine vibration signals). Another more mathematical application of the cepstrum is its use in calculating the minimum phase spectrum corresponding to a given log amplitude spectrum (i.e. a Hilbert transform). This could for example have application to loudspeakers, where minimum phase characteristics are often desired, and comparison between actual and ideal phase characteristics could be made.

Basic Theory

Using the terminology $\mathscr{F}\{\}$ to indicate the forward Fourier transform of the bracketed quantity, the original definition of the cepstrum is:

$$c(\tau) = |\mathscr{F}\{\log F_{xx}(f)\}|^2 \tag{1}$$

where the power spectrum of the time signal $f_x(t)$ is given by:

$$F_{xx}(f) = |\mathscr{F}\{f_x(t)\}|^2 \tag{2}$$

The new definition of the power cepstrum is:

$$c_p(\tau) = \mathscr{F}^{-1}\{\log F_{xx}(f)\} \tag{3}$$

while the autocorrelation function is given by:

$$R_{xx}(\tau) = \mathscr{F}^{-1}\left\{ F_{xx}(f) \right\} \tag{4}$$

A further definition of the cepstrum found useful by the author is the "amplitude spectrum of the logarithmic spectrum" or:

$$c_a(\tau) = \left| \mathscr{F}\left\{ \log F_{xx}(f) \right\} \right| \tag{5}$$

This can be interpreted as the square root of Eqn.(1) or as the modulus of Eqn.(3), since for a real even function such as a log power spectrum, the forward and inverse transforms give the same result.

The complex cepstrum may be defined as follows:

$$c_c(\tau) = \mathscr{F}^{-1}\left\{ \log F_x(f) \right\} \tag{6}$$

where $F_x(f)$ is the complex spectrum of $f_x(t)$ i.e.

$$F_x(f) = \mathscr{F}\left\{ f_x(t) \right\} = a_x(f) + i\, b_x(f) = A_x(f)\, e^{i\phi_x(f)} \tag{7}$$

in terms of real and imaginary components or amplitude and phase, respectively.

From (7) the (complex) logarithm of $F_x(f)$ is given by

$$\log F_x(f) = \log A_x(f) + i\phi_x(f) \tag{8}$$

Where $f_x(t)$ is real, as is normally the case, then $F_x(f)$ is "conjugate even", i.e.

$$F_x(-f) = F_x^{*}(f) \tag{9}$$

from which the following relationships follow:

$$\begin{aligned} a_x(f) &\quad \text{is even} \\ b_x(f) &\quad \text{is odd} \\ A_x(f) &\quad \text{is even} \\ \log A_x(f) &\quad \text{is even} \\ \phi_x(f) &\quad \text{is odd} \end{aligned} \tag{10}$$

From (10) and (8) it follows that log $F_x(f)$ is also conjugate even, and thus its inverse transform, the complex cepstrum, is a real-valued function, despite its name.

It is to be noted that for calculation of the complex cepstrum, the phase function $\phi_x(f)$ must be continuous, rather than the principal values modulo 2π, and this "unwrapping" of the phase spectrum can present problems in many practical situations (Ref. [3]). It will also be seen from Eqn. (2) that $F_{xx}(f) = A^2{}_x(f)$ and thus the power cepstrum of Eqn. (3) is virtually the same as the complex cepstrum of Eqn. (6) for functions whose phase $\phi_x(f)$ is identically zero.

Another practical problem which arises, is the question as to whether the power spectrum should be one-sided or two-sided in frequency. In all cases involving editing and transformation in both directions the 2-sided spectra should presumably be used, but for some diagnostic applications it is found to be advantageous to use one-sided spectra (negative frequency components set to zero). The theoretical background for this is tied up with the theory of Hilbert transforms and so a brief introduction will be given.

From the general theory of Fourier transforms (e.g. Ref. [4]) it is known that the spectrum of a real even function is real and even, and of a real odd function is imaginary and odd. Since any real function can be divided into even and odd components it follows that the real part of the Fourier transform comes from the even part of the time signal, and the imaginary part from the odd part of the time signal.

In the particular case of a causal time signal (i.e. one equal to zero for negative time) a special situation arises. As illustrated in Fig.1 the even and odd components must be identical for positive time in order that they will cancel and give zero for negative time. Thus, the even and odd components are related by the sign function and the real and imaginary parts of the Fourier transform are no longer independent. The imaginary part can be obtained from the real part by convolution with the Fourier transform of the sign function, viz. a hyperbolic function in the imaginary plane. This convolution constitutes the (inverse) Hilbert transform, which is thus the relationship between the real and imaginary parts of the spectrum of a causal function (or more generally of any one-sided function).

In Ref. [4] it is shown that for minimum phase functions the log amplitude and phase spectra are also related by the Hilbert transform. It follows

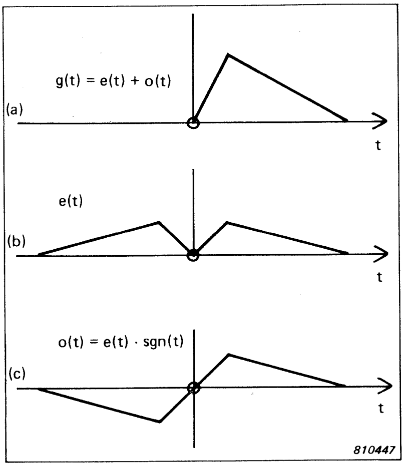


Fig. 1 Division of a causal function into even and odd components

directly that the time signal obtained by inverse transforming the complex spectrum having log amplitude as real part and phase as imaginary part (i.e. the complex cepstrum, see Eqns. (6) and (3)) must be causal, i.e. the complex cepstrum of minimum phase functions is right-sided only, and is identically zero for negative time (quefrency).

Thus one way of deriving the minimum phase spectrum corresponding to a given log amplitude spectrum is to first calculate the power cepstrum according to Equation (3). This is a real even function but can be considered as the even part of the (one-sided) complex cepstrum of the equivalent minimum phase function, which can thus easily be derived by doubling the positive quefrency values and setting the negative quefrency values to zero. A forward transform of this cepstrum will thus have the original log amplitude spectrum as real part, and the desired phase spectrum as imaginary part. An example of this for a loudspeaker is given later.

Returning to the question of whether the power cepstrum should be obtained from a one-sided or two-sided power spectrum, reference can be made to Fig.2. This shows the result of forward transforming a one-sided spectrum. The real part of this transform comes from the even part of the original function (by analogy with Fig.1) and thus the true cepstrum of the two-sided spectrum can be simply obtained by doubling the real part and discarding the imaginary part (see Appendix B1). On the other hand, the imaginary part will be the Hilbert transform of the real part and

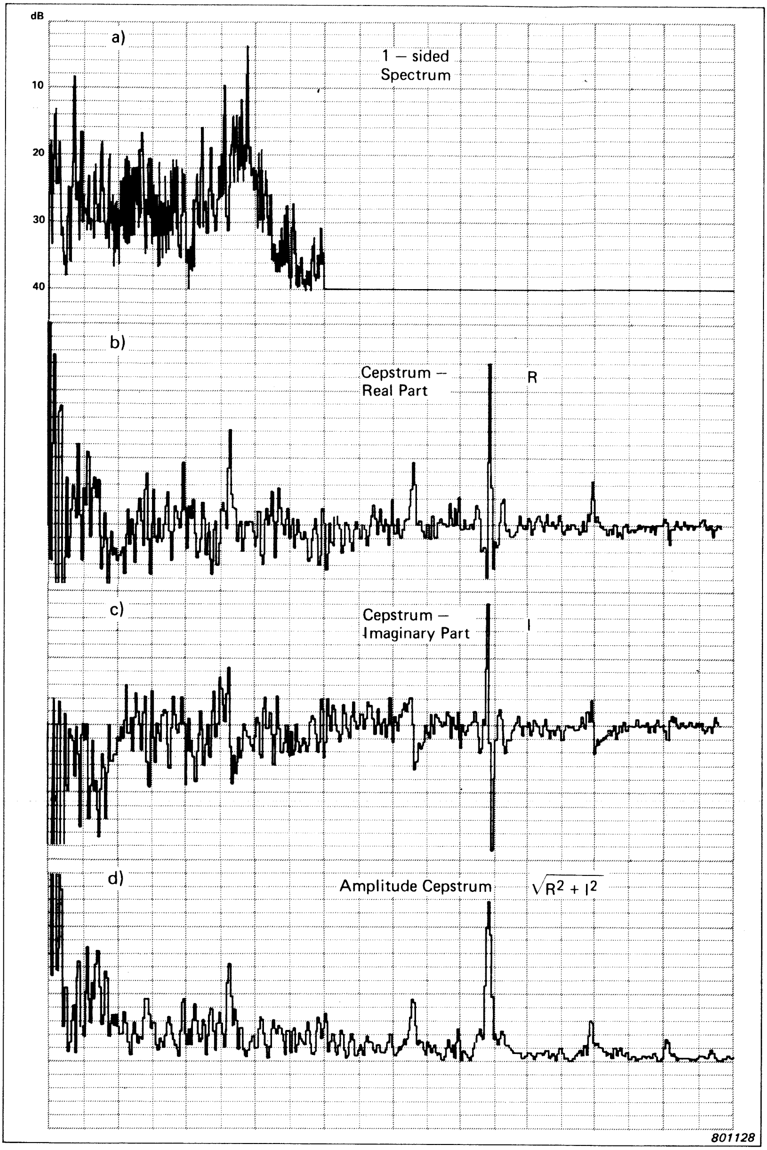


Fig. 2. Cepstrum calculation procedure for a one-sided spectrum

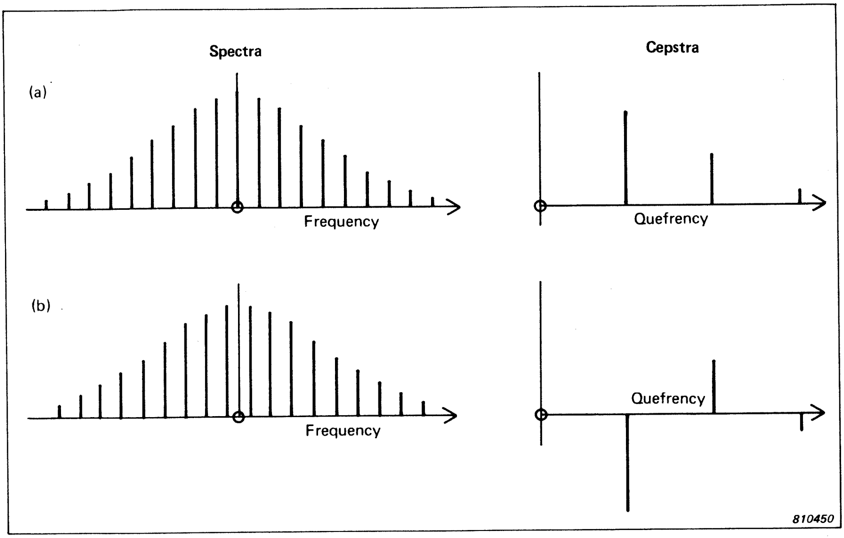


Fig. 3. (a) Cepstrum of a harmonic series
(b) Cepstrum of an odd harmonic series

thus has interesting properties. It will be noticed that in this case there are zero crossings in the imaginary part corresponding to the main peaks in the real part, these peaks in fact corresponding to the spacing of the sidebands in the original spectrum. There is a special reason why these peaks are positive and in the real plane in this particular case, and it is because the original sidebands fall at exact harmonic frequencies. Referring to Fig.3(a) this shows that the cepstrum of a harmonic series is a series of positive rahmonics. On the other hand if the periodic components in the spectrum are displaced a half spacing (e.g. a series of odd harmonics, see Fig.3(b)) the cepstrum consists of an alternating series of rahmonics with the first one negative. Halfway between these two situations the true cepstrum peak would be at right angles to the real plane and would thus correspond to a zero crossing in the real part of the cepstrum (but a peak in the imaginary part which is its Hilbert transform). To summarize, whenever the periodic structure of the spectrum does not correspond to a true harmonic series (in principle including zero frequency) the best version of the cepstrum to use is the amplitude cepstrum of the one-sided spectrum, corresponding to Fig.2(d). The peak in this function will always indicate the true spacing of components in the spectrum, independent of their displacement along the frequency axis. A situation where this is relevant is in the calculation of the cepstra of

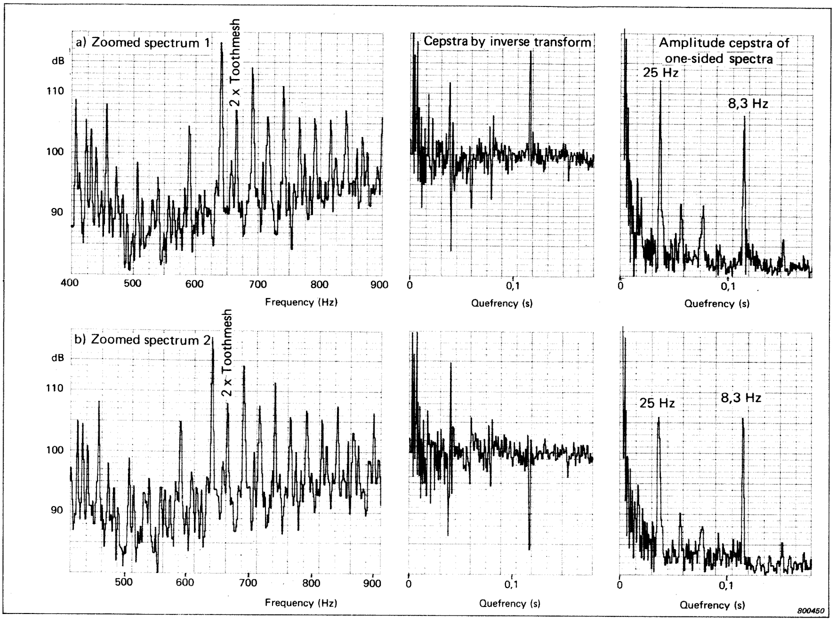


Fig. 4. Cepstra of slightly displaced zoom spectra

"zoom" spectra where the lower limiting frequency of the zoom range is interpreted as being zero frequency. Fig.4 shows an example of cepstra calculated from slightly displaced zoom spectra from the same signal. The cepstra calculated according to Eqn. (3) vary considerably as a result of this slight displacement, whereas the amplitude cepstra of the one-sided spectra are much more similar and easy to interpret. On the other hand it would not be possible to edit in these amplitude cepstra and return to the power spectra.

Deconvolution

Several of the applications of the cepstrum involve deconvolution, so the background for this will now be explained. Fig.5 shows schematically how the output signal $f_y(t)$ from a physical system can be considered as the convolution of the input signal $f_x(t)$ and the impulse response $h(t)$ of the system.

$$f_y(t) = f_x(t) * h(t) \qquad (11)$$

By the convolution theorem, this transforms to a multiplication in the frequency domain, i.e.

$$F_y(f) = F_x(f) \cdot H(f) \qquad (12)$$

and by taking logarithms this multiplication transforms to a sum:

$$\text{i.e.} \quad \log F_y(f) = \log F_x(f) + \log H(f) \qquad (13)$$

Because of the linearity of the Fourier transform, this additive relationship is maintained in the (complex) cepstrum

$$\text{i.e.} \quad \mathscr{F}^{-1}\left\{\log F_y\right\} = \mathscr{F}^{-1}\left\{\log F_x\right\} + \mathscr{F}^{-1}\left\{\log H\right\} \qquad (14)$$

Note that by squaring the amplitudes in Eqn.(12) we obtain:

$$F_{yy}(f) = F_{xx}(f) \cdot |H(f)|^2 \qquad (15)$$

and so the same additive relationship also applies to the power cepstrum:

$$\text{i.e.} \quad \mathscr{F}^{-1}\{\log F_{yy}\} = \mathscr{F}^{-1}\{\log F_{xx}\} + \mathscr{F}^{-1}\{2 \log |H|\} \qquad (16)$$

The above means that if the effect of one of the factors (source or transmission path) is known in the cepstrum, then subtracting it there will result in a deconvolution in the time domain. As an example, it will later be shown that echoes give a series of delta functions at known locations in the cepstrum. By subtracting these from the cepstrum all information about the echoes is removed, and by transformation back to the spectrum, and even to the time signal if the complex cepstrum has been used, the echoes will also be removed there.

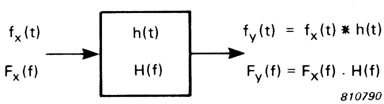


Fig. 5. Input - Output relations for a linear system

Note that the autocorrelation function would be obtained by inverse transformation of Eqn. (15), and that the multiplication there would transform to a convolution of the source and transmission path effects, as opposed to the additive relationship of the cepstrum. This represents one of the advantages of the cepstrum over the autocorrelation function. Another is that echoes are more readily detected in the cepstrum, in particular when the power spectrum is not flat, as pointed out in Ref. [1].

Terminology

The name "**ceps**trum" is derived by paraphrasing the word "**spec**trum" and was proposed in the original paper (Ref. [1]) along with a number of similarly derived terms. The reason was presumably that the cepstrum is a spectrum of a spectrum, but of course the same applies to the autocorrelation function, and the really distinctive feature of the cepstrum is the logarithmic conversion of the original spectrum.

Even so, many of the original terms are still found in the cepstrum literature, and so a short list will be given here of the most common:

Cepstrum	from	Spectrum
Quefrency	from	Frequency
Rahmonics	from	Harmonics
Gamnitude	from	Magnitude (amplitude)
Saphe	from	Phase
Lifter	from	Filter
Short-pass Lifter	from	Low-pass Filter
Long-pass Lifter	from	High-pass Filter

In the author's opinion, not all the above terms are necessary, or useful, but some are. For example, the quefrency is now well established as the X-axis of the cepstrum, even though it is identical with time. In principle there is no difference between quefrency and the "τ" of the autocorrelation function. Even so it is useful to speak of a "high quefrency" as representing rapid fluctuations in the spectrum (i.e. small frequency spacings) and "low quefrency" for more gentle variations (large frequency spacings). It can also be useful to distinguish between the actual "rahmonics" in the cepstrum and the effect in the cepstrum of a family of harmonics in the spectrum.

Applications

Echo detection and Removal

The application of the cepstrum to echo detection and removal is based on the fact that echoes give a periodic structure to the spectrum which in turn becomes a series of delta functions in the cepstrum, which are easy to locate and remove. Even in the case of non-ideal reflections it will be shown that instead of a delta function, the cepstrum contains the impulse response of the reflection, and can thus be used to measure the properties of a reflecting surface.

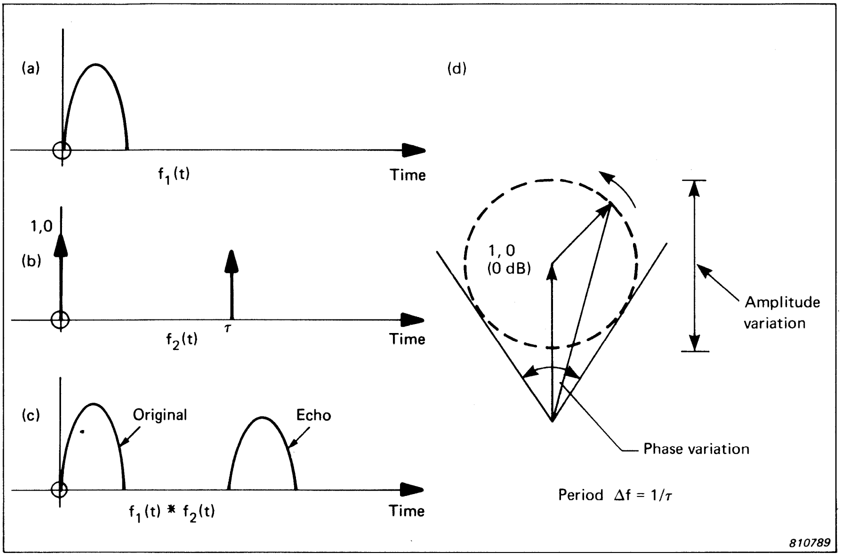


Fig. 6. Modelling a signal with echo as a convolution

Starting with the case of an ideal reflection, it will be seen with reference to Fig.6 that a signal containing an echo can be modelled as the convolution of the original signal with a pair of delta functions, one of unit area at time zero, and one at the echo delay time with reduced weighting, corresponding to the attenuation of the echo.

This convolution in time corresponds in frequency to a multiplication of their respective Fourier spectra (by the convolution theorem) and on taking the logarithm of the spectrum the effect of the echo becomes additive. The spectrum of the pair of delta functions can be derived intuitively by considering the analogous case of a spectrum consisting of two such delta functions and determining the corresponding time signal (by virtue of the symmetry of the Fourier transform). As illustrated in Fig.6(d), it could be considered as a unit DC component (i.e. zero dB) with a smaller additive rotating vector rotating with a period of $1/\tau$, where τ is the echo delay time. The resultant will be seen to have periodically varying amplitude around zero dB and periodically varying phase around zero, the periodicity in both cases being at intervals of $1/\tau$ in the spectrum. The effect of a forward rather than an inverse transform is only to reverse the direction of rotation of the vector.

This periodicity in the (logarithmic) spectrum transforms by an inverse

Fourier transform to a series of rahmonics in the cepstrum (either power cepstrum or complex cepstrum) with a spacing equal to τ. Thus, the existence and delay time of echoes is much easier to establish in the cepstrum than in the autocorrelation function, where even perfect reflections would give a scaled-down version of the autocorrelation of the original signal at the echo delay time. Only in the limiting case of white noise would this be a delta function. Thus, the cepstrum is much less sensitive to the shape of the power spectrum, and this can of course be considered a direct result of the logarithmic conversion.

The other advantage of the cepstrum over the autocorrelation function is that the effect of the echoes is additive, and thus subtracting the delta functions from the cepstrum removes the effect of the echoes completely.

The use of the cepstrum for locating echoes and measuring their delay time has been found useful in seismological investigations and underwater measurements (including marine seismology) and is described inter al. in Refs.[5 - 11]. Some of these references also describe the use of the complex cepstrum for removing all echoes in seismic signals and recovering the original wavelet.

An example will be given here of the use of the cepstrum for removing the effect of an echo from the power spectrum of an acoustic signal from a loudspeaker with a reflecting surface behind the microphone. The instru-

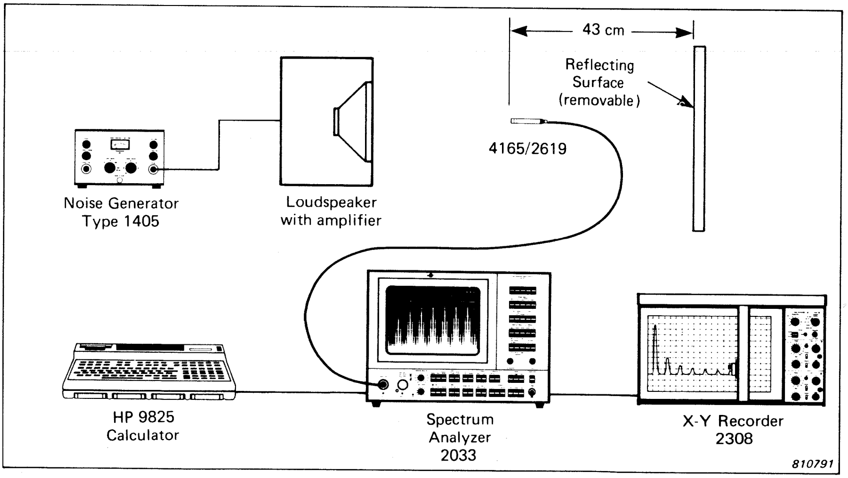


Fig. 7. Instrument set-up for echo removal

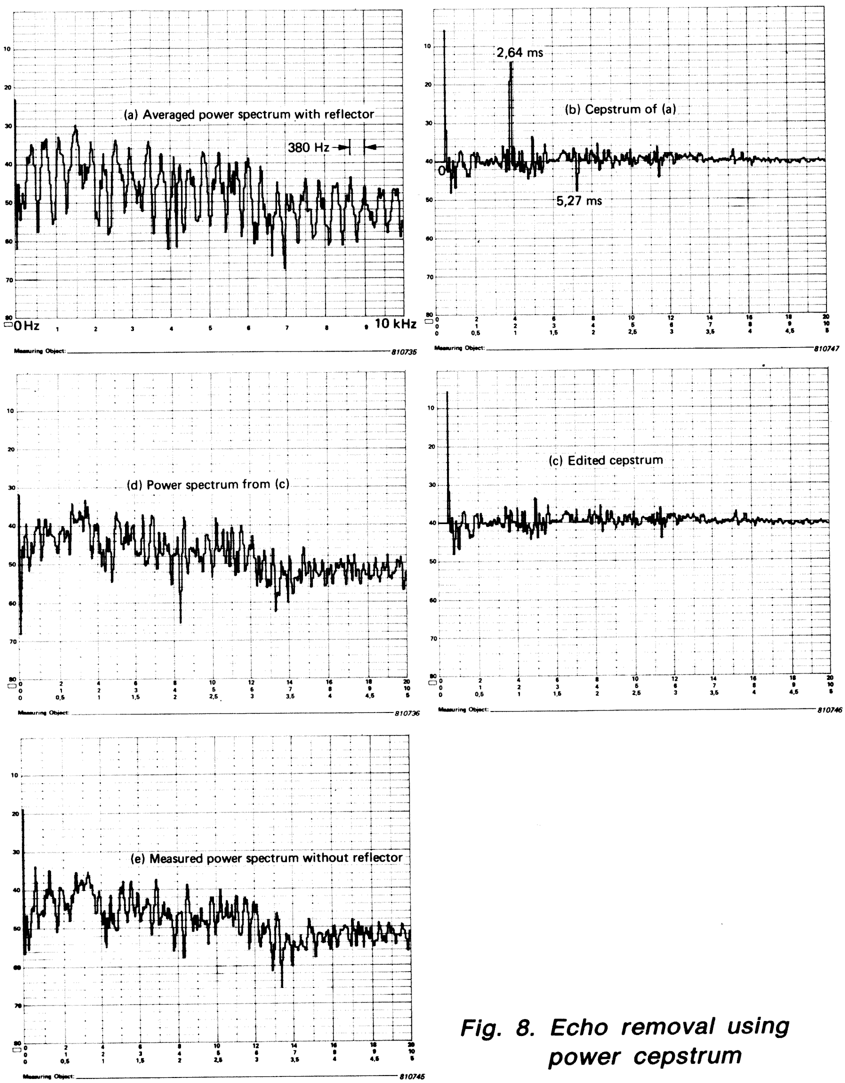
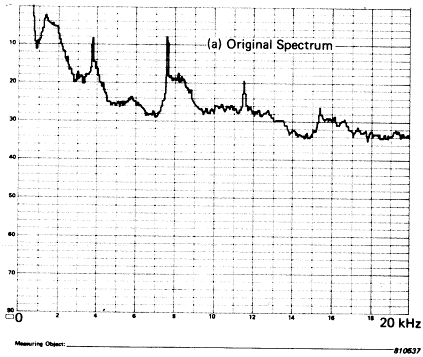
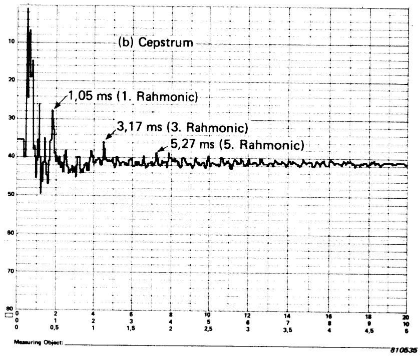


Fig. 8. Echo removal using power cepstrum

ment set-up is shown in Fig.7 and the measurement results in Fig.8. Fig.8(a) shows the averaged power spectrum of the signal received at the microphone when the loudspeaker is supplied with a white noise signal at the input. The periodic structure of the spectrum is clearly seen, and the spacing of 380 Hz corresponds well with the 2,64 ms delay time of the echo (the extra path of the reflected signal being 86 cm). Fig.8(b) shows the power cepstrum corresponding to Fig.8(a) and here the periodicity is transformed to a series of rahmonics with a spacing of 2,64 ms corresponding to the echo delay time. Fig.8(c) shows the effect in the cepstrum of removing the rahmonics with a coarse "comb lifter" (setting the cepstrum to zero at these points). A better approach would be to interpolate over the rahmonics (Ref. [3]) or at least to use a smoother comb lifter (Refs.[12, 13]). Fig.8(d) shows the spectrum resulting from a forward transform of the edited cepstrum of Fig.8(c) and this can be compared with the spectrum of Fig.8(e) obtained with the reflecting surface removed. Even though a very coarse comb lifter was used, the spectra are very similar.

A practical problem of this type to which the cepstrum has been applied is the measurement of aero engine noise on the ground, where ground reflections become mixed with the direct signal. Fig.9 shows the results of some actual measurements in a case where the delay time was known to





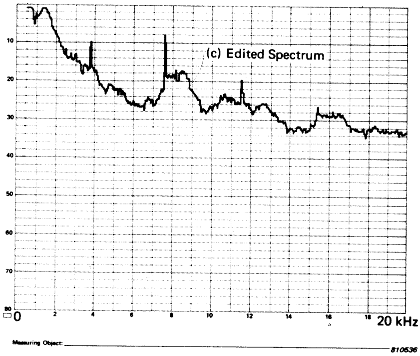


Fig. 9. Removal of ground reflections from aero engine noise

(a) Original power spectrum
(b) Cepstrum showing rahmonics of delay time 1,05 ms
(c) Power spectrum from edited cepstrum

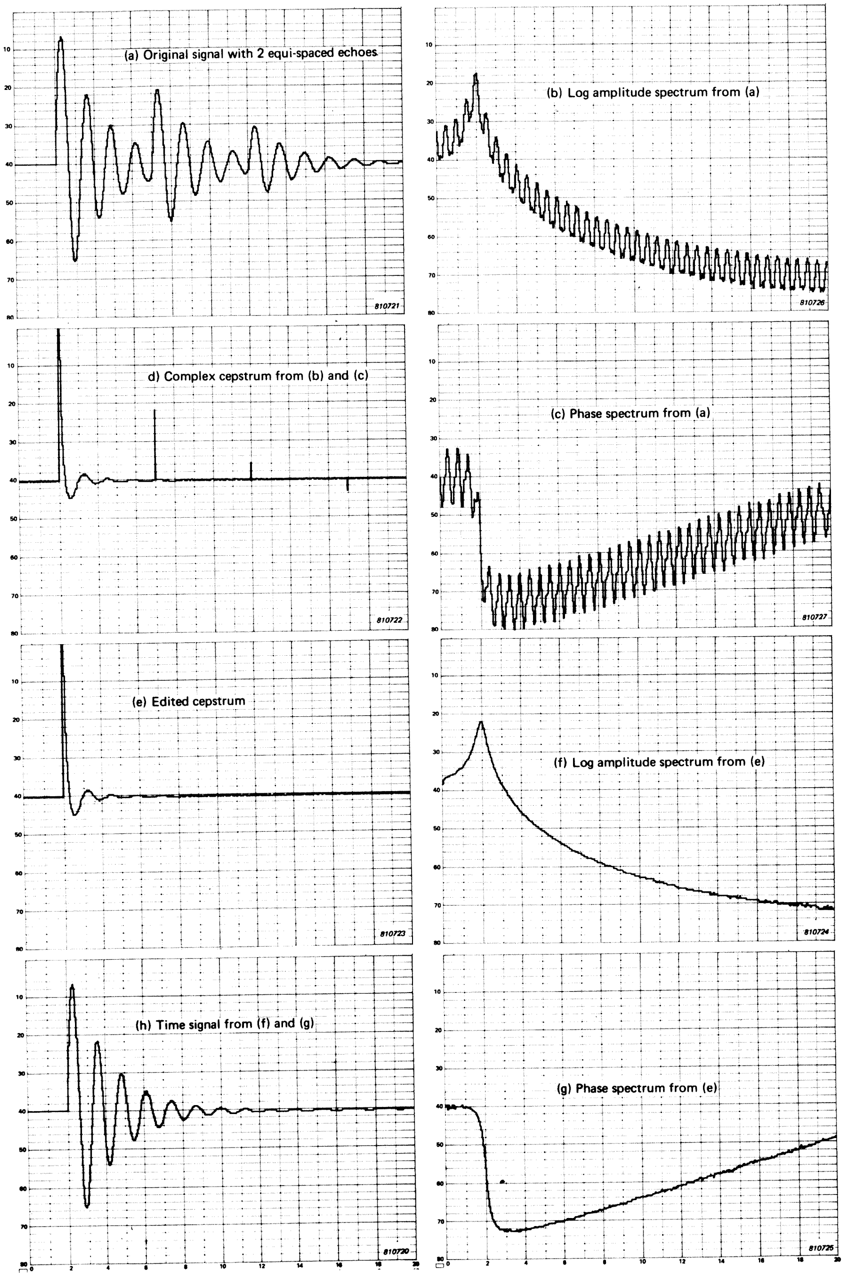


Fig. 10. Echo removal using complex cepstrum

be approx. 1,1 ms. Removal of the 1st, 3rd and 5th rahmonics of this quefrency in the (power) cepstrum results in quite a different spectrum shape. Ref. [14] discusses the potential of this technique, based on a successful preliminary investigation.

In Ref. [13] a similar procedure is used to measure the amplitude characteristic of a loudspeaker in a normal room, obtaining virtually the same results as in an anechoic chamber. Impulse excitation was used rather than noise. The reflections from the three pairs of walls were edited away in the cepstrum using three comb lifters adapted to the three reflection times. Very good results were obtained as low as 50 Hz, whereas editing of the reflections directly in the time signal only gave good results down to 1000 Hz.

A similar procedure can be used in the complex cepstrum to obtain both amplitude and phase characteristics of a loudspeaker. A numerically generated example is given in Fig.10 to illustrate the basic principles. Even though the reflection overlaps the original signal, the delta functions in the complex cepstrum are reasonably well removed from the cepstrum of the basic signal, which is much shorter than the original signal (because the logarithmic spectrum is much flatter than the direct Fourier spectrum). It is thus relatively simple to remove the effects of the echo from the cepstrum, and transforming all the way back to the time signal shows that the removal has been quite efficient.

Fig.11 shows the result of applying the same technique on an actual loudspeaker excited by a 50 μs square pulse in a normal room. The (first part of the) time signal in Fig.11(a) shows a number of reflections starting at about 6,2 ms delay time. In contrast to the numerically generated example of Fig.10, the phase spectrum had to be "unwrapped" and scaled down appropriately before calculation of the complex cepstrum, which is shown in Fig.11(b). The effect of the reflections in the cepstrum is seen to be rather spread out, indicating non-ideal reflections (see next section) and so a fairly drastic "short-pass lifter" was applied to remove all reflections (a Hanning window falling to zero at 5 ms). The resulting log amplitude and phase characteristics (to 10 kHz) are shown in Fig.11(c). For comparison purposes, a measurement made on the same loudspeaker using the TDS (Time Delay Spectrometry) technique (Ref.[15])is shown in Fig.12. Even though this was made in a different room, the similarities are quite striking.

At the same time as the 10 kHz results of Fig.11 another measurement was made using the cepstrum technique but with an upper limiting

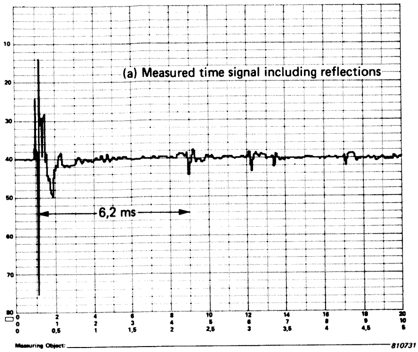
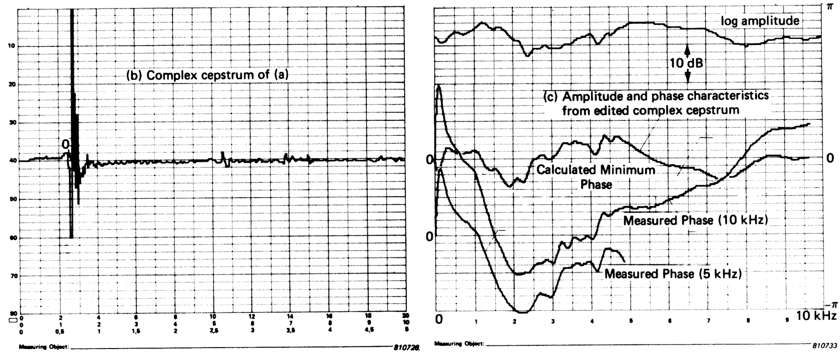


Fig. 11. Measurement of loudspeaker characteristics in a hard room using complex cepstrum



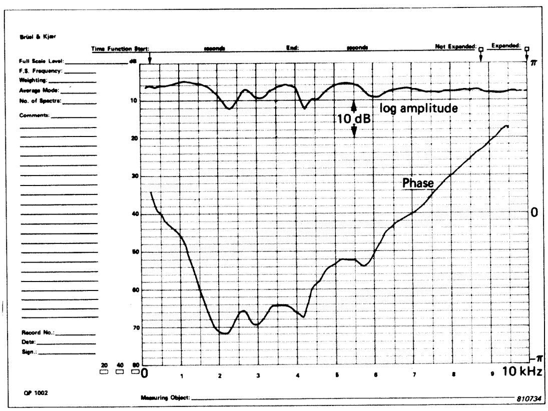


Fig. 12. Loudspeaker characteristics measured using TDS system

frequency of 5 kHz. Because of the double length of record, the added noise (and reflections) caused problems with the phase unwrapping algorithm. There were three places in the spectrum where the algorithm "jumped" the wrong way, introducing a discontinuity. In this case it was possible to solve the problem by applying a decaying exponential window to the original time signal, thus considerably reducing the effect of noise (and reflections) at the end of the record. After removal of the reflections in the cepstrum, the signal was transformed all the way back to the impulse response which was compensated for the exponential window before forward transforming again to obtain the amplitude and phase characteristics. The phase result is drawn in Fig.11(c) on the same scale (but with twice the resolution).

The application of the complex cepstrum to loudspeakers is one case where the phase unwrapping is not too difficult because the phase spectrum should be smooth and the amplitude spectrum should not contain zeroes. For more general signals, the phase unwrapping can present considerable problems (Ref. [3]).

As a matter of interest, Fig.11(c) also includes the minimum phase characteristic corresponding to the measured log amplitude characteristic (to 10 kHz), calculated as outlined previously.

Measurement of the properties of a reflecting surface

The following derivation is taken basically from Ref. [16] where the cepstrum is proposed as a technique for measurement of the absorption properties of a partially reflecting surface.

Consider the case depicted in Fig.13 where the signal y(t) received at a microphone is the sum of the direct signal x(t) and a reflected signal modified by the reflecting surface and attenuated because of the longer path.

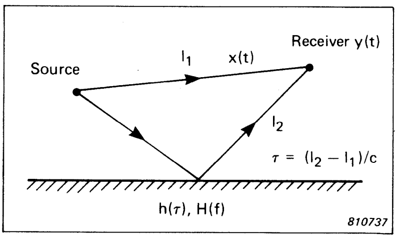


Fig. 13. Signal with a non-ideal reflection

Thus, in the time domain

$$y(t) = x(t) + \frac{l_1}{l_2} x(t) \ast h(t-\tau) \tag{17}$$

Transforming this equation by the Fourier transform gives

$$Y(f) = X(f)\left[1 + \frac{l_1}{l_2} H(f)e^{-i2\pi f\tau}\right] \tag{18}$$

The power spectrum can be obtained as the modulus squared

$$|Y(f)|^2 = |X(f)|^2 \left[1 + \frac{l_1}{l_2} H\, e^{-i2\pi f\tau}\right]\left[1 + \frac{l_1}{l_2} H\, e^{-i2\pi f\tau}\right]^{*} \tag{19}$$

from which the log power spectrum is:

$$\log|Y|^2 = \log|X|^2 + \log\left[1 + \frac{l_1}{l_2} H\, e^{-i2\pi f\tau}\right] + \log\left[1 + \frac{l_1}{l_2} H^{*}\, e^{+i2\pi f\tau}\right] \tag{20}$$

Expanding the log (1 + x) terms as $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\ldots$

and inverse transforming to the power cepstrum gives

$$C_y(t) = C_x(t) + \frac{l_1}{l_2} h(t-\tau) - \left(\frac{l_1}{l_2}\right)^2 \frac{1}{2} h(t-\tau) \ast h(t-\tau) + \ldots\ldots$$

$$+ \frac{l_1}{l_2} h(-t-\tau) - \left(\frac{l_1}{l_2}\right)^2 \frac{1}{2} h(-t-\tau) \ast h(-t-\tau) + \ldots\ldots \tag{21}$$

meaning that the impulse response of the reflection h(t) will be found in the power cepstrum delayed by the echo delay time τ and scaled down by the attenuation factor l_1/l_2. At the higher rahmonic quefrencies of the delay time the impulse response is convolved with itself progressively more. At negative quefrencies the mirror image is found, because the power cepstrum is an even function.

Thus, provided both the cepstrum of the original signal, and the impulse response of the reflection are shorter than the delay time τ, it should be possible to extract the impulse response relatively easily from the overall cepstrum. It is interesting that this impulse response may be Fourier transformed to give both the amplitude and phase characteristics of the reflection, even though the power cepstrum has been used. It is also interesting that in the special case of a perfect reflection with no amplitude or phase distortion, and no variation with frequency, the impulse response will of course be a delta function, as predicted previously.

Ref. [16] discusses some of the practical points involved in making such measurements.

Speech Analysis

The applications of the cepstrum to speech analysis are mainly connected with its ability to separate source and transmission path effects, provided they have different quefrency contents. This is usually the case with speech where the source spectrum, i.e. of the voice, is very flat, contain-

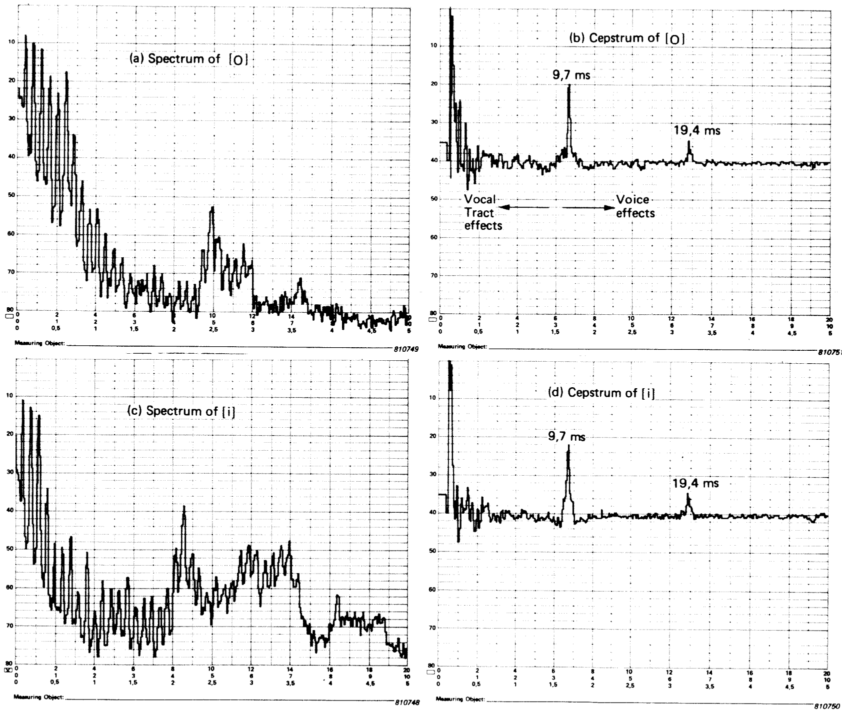


Fig. 14. Spectra and cepstra for two vowels

ing a large number of harmonics of the voice pitch, but is modified by the resonance characteristics of the vocal tract, the so-called formants, which determine for example which vowel is being uttered. Fig.14 shows spectra and cepstra for the vowels "oh" [o] and "ee" [i] and illustrates how the differences mainly lie in the low quefrency part of the cepstrum, which is dominated by the formant characteristic. Non-voiced sounds, such as many consonants and whispered speech, do not give peaks in the cepstrum corresponding to the voice pitch, and one of the earliest applications of the cepstrum was to separate voiced and non-voiced sounds, and to measure voice pitch (Refs.[17, 18]).

It is also possible by editing in the cepstrum to remove one effect completely, for example the voice, and thus simplify the tracking of the formants. Fig.15 from Ref. [19] shows a typical situation, a 3-dimensional representation of the section "ea" from the word "Montreal". The picture is quite confused, but by "shortpass liftering" each of the spectra to remove the voice components, as shown in Figs.16 and 17, only the formants are left, and the picture becomes much clearer.

In Refs.[20, 21] it is also described how the cepstrum can be used for efficient vocoding and transmission of speech. Most of the intelligence in

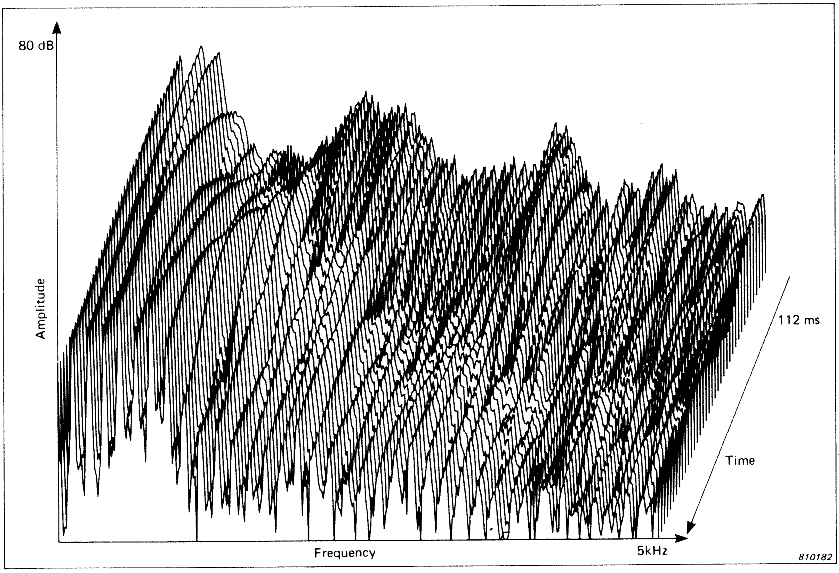


Fig. 15. Scan spectrum of "ea" in "Montreal"

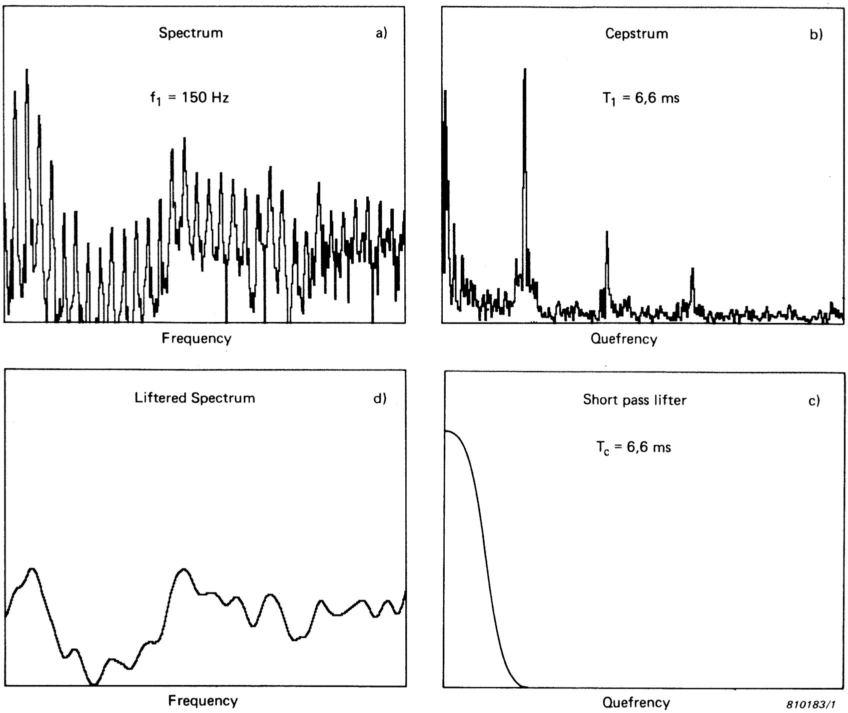


Fig. 16. Cepstrum liftering
a) log power spectrum of vowel
b) magnitude of cepstrum
c) short pass lifter characteristic
d) short pass liftered log power spectrum

the speech is contained in the low quefrency part of the cepstrum, so only this is transmitted, along with information as to whether the speech is voiced, and if so, the voice pitch. At the receiver end, the speech is reconstituted using the low quefrency information to generate a filter characteristic (or impulse response) for a source which would éither be a variable frequency pulse generator, for the voiced sections, or a noise generator for the unvoiced sections. Despite the synthetic voice, the speech was reported as sounding natural.

As another application of the cepstrum, Ref. [22] shows how it can be useful to include it, along with spectral and other information, in pattern recognition algorithms for speaker identification. Inclusion of the cepstral information improved the ability of the technique to exclude impostors.

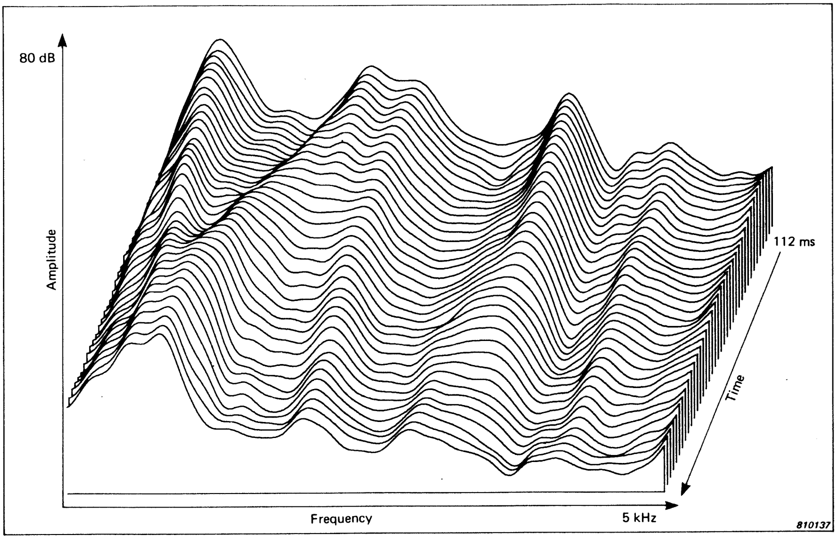


Fig. 17. Short pass liftered scan spectrum of "ea" in "Montreal"

Machine Diagnostics

The applications of the cepstrum to machine diagnosis are mainly based on its ability to detect periodicity in the spectrum, e.g. families of harmonics and uniformly spaced sidebands, while being insensitive to the transmission path of the signal from an internal source to an external measurement point.

In Ref. [23] the cepstrum is proposed as a technique to aid detection of missing blades in turbines, as it is shown there that such blade anomalies give rise to a large number of harmonics of the shaft rotational speed in measurements made both internally and externally on the casing in the vicinity of the affected blade row. Even though the harmonic pattern can be seen by eye, the whole family of harmonics is reduced in the cepstrum basically to one component, which is much easier to monitor.

A very similar reasoning is applicable to gearbox diagnosis, since tooth anomalies have a very similar influence on gearbox vibration signals as do blading anomalies on turbine signals (Ref. [24]). In Ref. [24] a very detailed discussion is given of the application of cepstrum analysis to gearbox diagnosis and so here the discussion will be limited to a couple of typical examples.

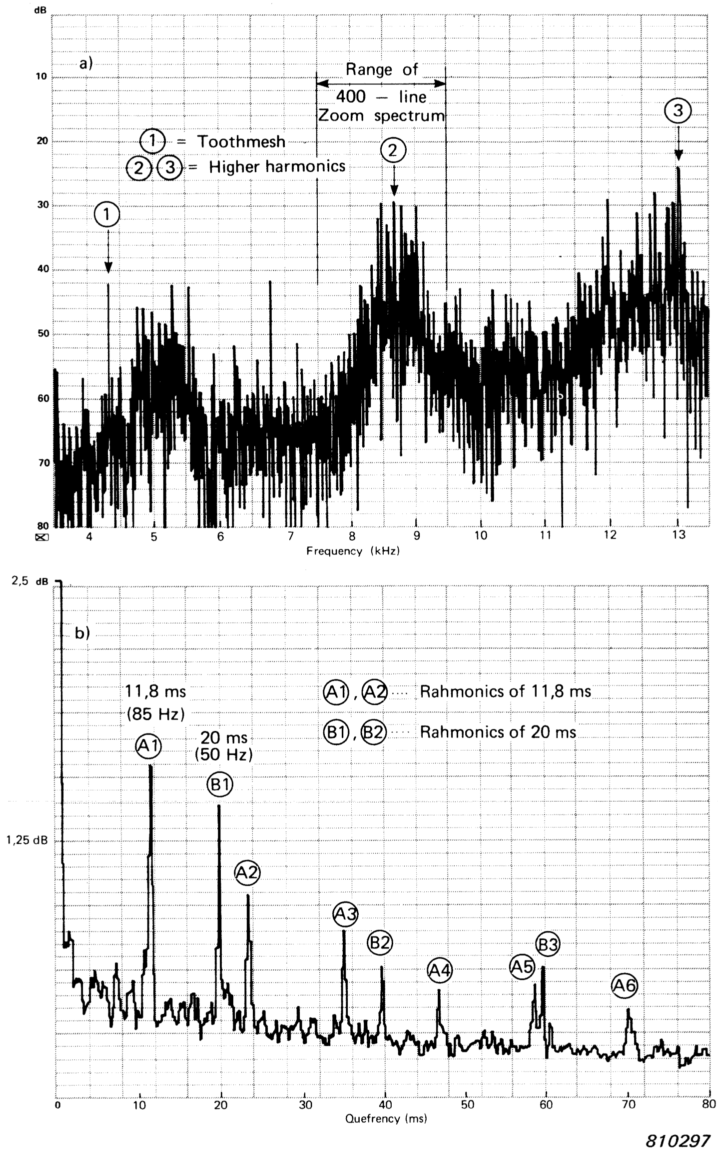


Fig. 18. Example of a cepstrum analysis on a gearbox vibration signal
(a) 2000-line logarithmic power spectrum (3,5 – 13,5 kHz)
(b) Average cepstrum calculated from 5 such spectra

In gearbox vibrations any deviations from exact uniformity of each toothmesh tend to show up partly as harmonics of the shaft speed and also as sidebands around the toothmeshing harmonics caused by modulation of the toothmesh signal by the lower rotational frequencies. The sideband spacing thus contains valuable information as to the source of the modulation and can be extracted using the cepstrum. The cepstrum has the two advantages of being able to detect periodicity not immediately apparent to the eye, and of being able to measure it very accurately because it gives the average sideband spacing over the whole spectrum.

Fig. 18 illustrates the first advantage and was made using an FFT Analyzer Type 2033 in conjunction with an HP 9825 desktop calculator. Fig. 18(a) is a 2000-line spectrum which includes the first three harmonics of the toothmeshing frequency of a single reduction gearbox. It purposely excludes the low harmonics of the shaft speeds since these may have other causes than the toothmeshing. The spectrum was obtained by performing five 400-line zoom analyses (on the same data) and storing the intermediate results in the calculator memory. The 2000-line spectrum was then read digitally back into the 10 K input memory of the analyzer and frequency analyzed once more (using the "scan average" procedure with 75% overlapping Hanning windows) to obtain the cepstrum. Fig. 18(b) actually represents the average of five such cepstra. Even though it is difficult to see any periodic structure in the spectrum, it is apparent from the cepstrum that there are two families of sidebands with spacings of 85 Hz and 50 Hz respectively, the rotational speeds of the two gears. All significant components in the cepstrum stem from one or other of these two shaft speeds.

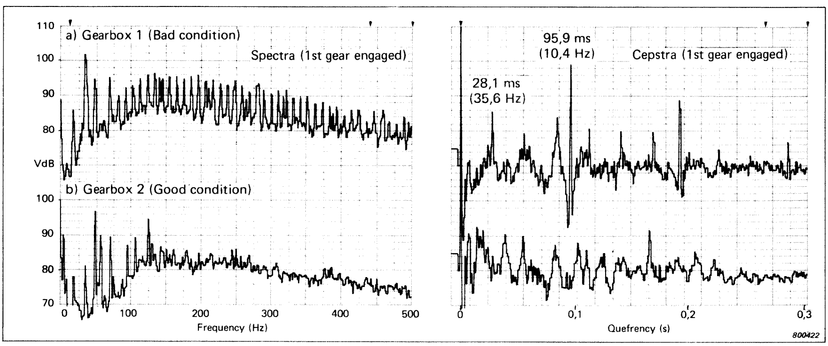


Fig. 19. Spectra and cepstra from truck gearboxes in good and bad condition

Fig. 19 illustrates the other advantage, and shows spectra and cepstra for two truck gearboxes, in good and bad condition respectively, running on a test stand. The good gearbox shows no marked spectrum periodicity, but the spectrum of the bad one contains a large number of sidebands with a spacing of approximately 10 Hz. The cepstrum gives this spacing very accurately as 10,4 Hz and thus excludes the possibility that it was the second harmonic of the output shaft speed 5,4 Hz. It was in fact traced to the rotational speed of second gear, even though this was idling because first gear was engaged.

As a matter of interest, Fig. 18 represents an example of the cepstrum of Eqn.(5), on a one-sided spectrum, while Fig. 19 uses Eqn.(3), effectively on a two-sided spectrum, although the procedure of Appendix B2 was used.

References

[1] BOGERT, B.P., HEALY, M.J.R. & TUKEY, J.W., "The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo-Autocovariance, Cross-cepstrum and Saphe Cracking", in *Proceedings of Symposium on Time Series Analysis* by Rosenblatt, M., (Ed.), Wiley, N.Y. 1963, pp. 209—243.

[2] OPPENHEIM, A.V., SCHAFER, R.W. & STOCKHAM, T.G. "Nonlinear Filtering of Multiplied and Convolved Signals", *IEEE Trans. Audio & Electroacoustics,* Vol. AU-16, No.3, Sept. 1968.

[3] CHILDERS, D.G., SKINNER, D.P. & KEMERAIT, R.C., "The Cepstrum: A Guide to Processing". *IEEE Proc.* Vol. 65, No.10, October 1977, pp. 1428—1443.

[4] PAPOULIS, A., "The Fourier Integral and its Applications", McGraw-Hill, 1962.

[5] COHEN, T., "Source-depth determinations using spectral, pseudo-autocorrelation and cepstral analysis", *Geophys. J. Roy. Astron. Soc.,* Vol. 20, pp. 223—231, 1970.

[6] FJELL, P.O., "Use of the cepstrum method for arrival times extraction of overlapping signals due to multipath conditions in shallow water", *J. Acoust. Soc. Amer.,* Vol.59, no. 1, pp. 209—211, Jan. 1976.

[7] BUTTKUS, B., "Homomorphic Filtering – Theory and Practice". *Geophysical Prospecting* Vol.23, pp. 712—748, 1975.

[8] STOFFA, P.L., BUHL, P. & BRYAN, G.M. "The application of homomorphic deconvolution to shallow-water marine seismology — Part I: Models", *Geophysics,* Vol.39, no.4, pp. 401—416, Aug. 1974.

[9] BUHL, P., STOFFA, P.L. & BRYAN, G.M. "The application of homomorphic deconvolution to shallow-water marine seismology – Part II: Real data," *Geophysics,* vol.39, no.4, pp. 417–426, Aug. 1974.

[10] HASSAB, J.C., "Homomorphic Deconvolution in Reverberant and Distortion Channels: An Analysis". *J. Sound and Vibration,* Vol.58, No.2, pp. 215—231, 1978.

[11] ULRYCH, T.J., "Application of Homomorphic Deconvolution to Seismology", *Geophys.,* Vol.36, No.4, pp. 650—660. Aug. 1971,

[12] OOYAMA, G., KATAGIRI, S. & KIDO, K., "A New Method of Cepstrum Analysis by using Comb Lifter" *1978 IEEE Conf. ASSP,* Tulsa.

[13] NITTA, T. & TANAKA, M., "Free-field Measurements for a loudspeaker System in a Normal Room – Using Digital Signal Processing Techniques". *1978 IEEE Conf. ASSP,* Tulsa.

[14] SYED, A.A., BROWN, J.D., OLIVER, M.J. & HILLS, S.A., "The Cepstrum: A Viable Method for the Removal of Ground Reflections." *J. Sound and Vibration,* Vol.71, No.2, pp. 299—313, 1980

[15] BIERING, H. & PEDERSEN, O.Z., "Free-Field Techniques. A comparison of time selective free-field techniques with respect to frequency resolution, measuring time, and noise-induced errors." Paper presented at 68th. A.E.S. Convention, Hamburg 1981. Available from Brüel & Kjær, Nærum, Denmark.

[16] BOLTON, J.S. & GOLD, E. "The Application of Cepstral Techniques to the Measurement of Reflection Coefficients in situ. Part 1: Simulations." *R.M.I.T. Report 121007.* Royal Melbourne Institute of Technology, Melbourne, Australia, 1980.

[17] NOLL, A.M. "Cepstrum Pitch Determination", *J.A.S.A.* Vol.41, No.2, pp. 293—309. 1967,

[18] SCHAFER, R.W. & RABINER, L.R., "Digital Representations of Speech Signals." *Proc. IEEE* Vol.63, pp. 662—677, 1975.

[19] THRANE, N. "Application of a Long Memory FFT Analyzer in Speech Analysis." Paper presented at the Fourth F.A.S.E. Symposium on Acoustics and Speech, Venezia, Italy, 1981. Available as Brüel & Kjær Application Note No. 066–81.

[20] SCHAFER, R.W. & RABINER, L.R. "System for Automatic Formant Analysis of Voiced Speech." *J.A.S.A.* Vol.47, pp. 634—648, 1970.

[21] OPPENHEIM, A.V. "Speech Analysis–Synthesis System Based on Homomorphic Filtering." *J.A.S.A.* Vol.45, pp.458–465, 1969.

[22] LUCK, J.E. "Automatic Speaker Verification using Cepstral Measurements." *J.A.S.A.* Vol.46, pp. 1026—1032, 1969.

[23] SAPY, G. "Une application du traitement numérique des signaux au diagnostic vibratoire de panne: La détection des ruptures d'aubes mobiles de turbines". *Automatisme* – Tome XX, No.10, pp. 392—399, October 1975,

[24] RANDALL, R.B. "Cepstrum Analysis and Gearbox Fault Diagnosis." Brüel & Kjær Application Note No.233–80.

[25] THRANE, N. "The Discrete Fourier Transformer and FFT Analyzers." *B & K Technical Review* No.1–1979.

APPENDIX A

Calculation of the Cepstrum

The aim of this appendix is to indicate how the various forms of the cepstrum can be calculated using the FFT Analyzer Type 2033 (or 2031) in conjunction with a desktop calculator. Even though the analyzer basically performs a forward transformation of 1024 real data points, the results can be modified in the calculator so as to obtain the inverse transform of up to 1024 real or complex values thus giving the possibility of calculating both power cepstra and complex cepstra. The actual algorithms used are somewhat more generally applicable, and so are detailed in Appendix B.

1. Power Cepstrum

The digital version of Eqn.(3) for the power cepstrum is as follows:

$$C_p(n) = \mathscr{F}^{-1}\{\log F_{xx}(k)\} \qquad (A1)$$

where n stands for n Δt (where Δt is the sampling interval) and thus indicates the time. n runs from 0 to 1023. Likewise k represents the frequency k Δf (where Δf is the line spacing in the frequency spectrum) and in principle also runs from 0 to 1023 even though only the values from 0 to 512 are calculated. Because of the implicit periodicity of all functions calculated by the FFT process (Ref. [25]) the values of k from 512 to 1024 also represent the negative frequency components (from −512 to 0) and can usually be derived from the positive frequency values.

Since $F_{xx}(k)$ is a real even function, the inverse transformation can be replaced by a forward transformation (Appendix B1).

In general only the one-sided power spectrum is given, and the simpler calculation method of Appendix B2 will be advantageous.

With this method, only the one-sided spectrum is transformed, and the real part of the transform gives the desired cepstrum.

Another advantage of this method is that the "envelope" cepstrum (amplitude cepstrum of the one-sided spectrum) of Fig.4 may be obtained at the same time. In fact the analyzer itself automatically calculates this and displays it as the "Instantaneous" spectrum, which can be viewed on a linear amplitude scale.

The formula for the envelope cepstrum is

$$C_e(n) = \left| \mathscr{F}^{-1}\{\log G(k)\} \right| \tag{A2}$$

where G(k) is the one-sided power spectrum.

2. Complex Cepstrum

Corresponding to Eqn.(6) the formula for the complex cepstrum is

$$C_c(n) = \mathscr{F}^{-1}\{\ln A_x(k) + i\,\phi_x(k)\} \tag{A3}$$

Because the logarithmic spectrum is a conjugate even function, the calculation method of Appendix B3 may be used. Note that the phase function $\phi_x(k)$ must be "unwrapped" to a continuous function of frequency, in place of the principal values modulo 2π which are calculated from the real and imaginary parts of the complex spectrum. Moreover, the log amplitude must be scaled in nepers (natural log of the amplitude ratio) in order to correspond to the radians of the phase spectrum.

3. Practical Problems

Since the analyzers in general are AC coupled, the DC (i.e. zero frequency) value in the power spectrum is not calculated. It is therefore necessary to insert a value before calculating the cepstrum. It will be found in practice that the best results are obtained by setting the zero frequency component equal to the value of the neighbouring line.

Moreover, since the FFT algorithm used in the Analyzers Types 2033 and 2031 is optimised for signals with no DC-component, it is advantageous to subtract the mean log spectrum value before calculating the cepstrum. This will optimise the signal/noise conditions in the cepstrum, and is particularly valuable when editing and transformation in both directions is to be performed.

In calculation of the complex cepstrum it is advisable before attempting to unwrap the phase spectrum to remove any simple delay, which gives a linear slope to the phase spectrum. This should be done to the maximum extent possible in the time signal before transformation, and then in the phase spectrum itself by varying the linear component until the number of "jumps" over 2π is minimised.

APPENDIX B

Calculation of Inverse Fourier Transform

The forward and inverse discrete Fourier transforms, as calculated by the FFT analyzers, are defined as

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-i \frac{2\pi}{N} kn} \qquad \text{(B1)}$$

and

$$x(n) = \sum_{k=0}^{N-1} X(k)\, e^{i \frac{2\pi}{N} kn} \qquad \text{(B2)}$$

respectively,

where $X(k)$ = the discrete complex spectrum
$x(n)$ = the sampled time function
N = Number of samples in the time record.

Properties of the Fourier Transform

The Fourier transform which is implemented in the analyzers Types 2033 and 2031 is designed to be used for forward transformation of real-valued time signals, but by utilising some of the properties of the Fourier Transform, as listed in Tables I and II, it can also be used for forward and inverse transformation of any complex signals. The inverse transformation of the three types of signals:

(1) Real-valued
(2) Real and even
(3) Conjugate even

is described in the following.

The results are sketched in Figs.B1, B2 and B3 where the vertical lines

Algorithm	Conditions
$\mathscr{F}^{-1}\{X(k)\} = \left(N \;\mathscr{F}\{X^*(k)\}\right)^*$	any X(k)
$\mathscr{F}^{-1}\{X(k)\} = N \;\mathscr{F}\{X^*(k)\}$	x(n) real
$\mathscr{F}^{-1}\{X(k)\} = N \;\mathscr{F}\{X(k)\}$	x(n) real and even
$\mathscr{F}^{-1}\{X(k)\} = \left(N \;\mathscr{F}\{X(k)\}\right)^*$	X(k) real

Table I

Time signal	Spectrum
real and even	real and even
real and odd	imag and odd
imag and even	imag and even
imag and odd	real and odd
real	conjugate even
conjugate even	real

Table II

indicate the result of the FFT calculation, and the solid lines the desired result. Note that zero is there shown in the centre of the diagram. During many of the actual operations, zero frequency or time will actually be located at the start of the record, but because of the periodicity of all functions the negative frequencies or times will be located in the second half of the record.

B1. Real-valued Spectrum

From Table I it follows that

$$\mathscr{F}^{-1}\{X(k)\} = N\,[\mathscr{F}\{X(k)\}]^* \qquad \text{(B3)}$$

The calculation procedure (for positive time) is then as follows:

a) Forward transform
b) Form complex conjugate
c) Multiply by N

The result for both positive and negative time is seen in Fig.B1.

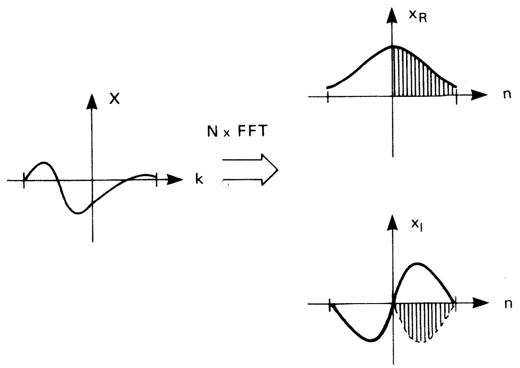


Fig. B1. Inverse transform of a real-valued spectrum

For the special case of even spectra it is possible to omit the step (b), but in that case the next procedure will normally be preferable anyway.

B2. Real and Even Spectrum

From the original symmetrical spectrum a new one-sided spectrum is formed which has the original spectrum as its even part and is equal to zero for negative frequencies. The real part of the inverse transform of such a spectrum is identical with the inverse transform of the original spectrum. Since normally only the positive frequency components of the original spectrum are given in any case, this saves forming the symmetrical spectrum for negative frequencies.

It follows that:

$$\mathcal{F}^{-1}\{X(k)\} = N\,R_e\,[\mathcal{F}\{\tilde{X}(k)\}] \qquad \text{(B4)}$$

where:

$$\tilde{X}(k) = \begin{cases} 2\,X(k), & 0 < k < 512 \\ X(k), & k = 0,\ k = 512 \\ 0, & -512 < k < 0 \end{cases}$$

$$\tilde{X}_e(k) = X(k)$$

The calculation procedure is thus as follows:

a) Form $\tilde{X}(k)$
b) Forward transform
c) Extract and scale the real part.

Fig.B2 shows the result.

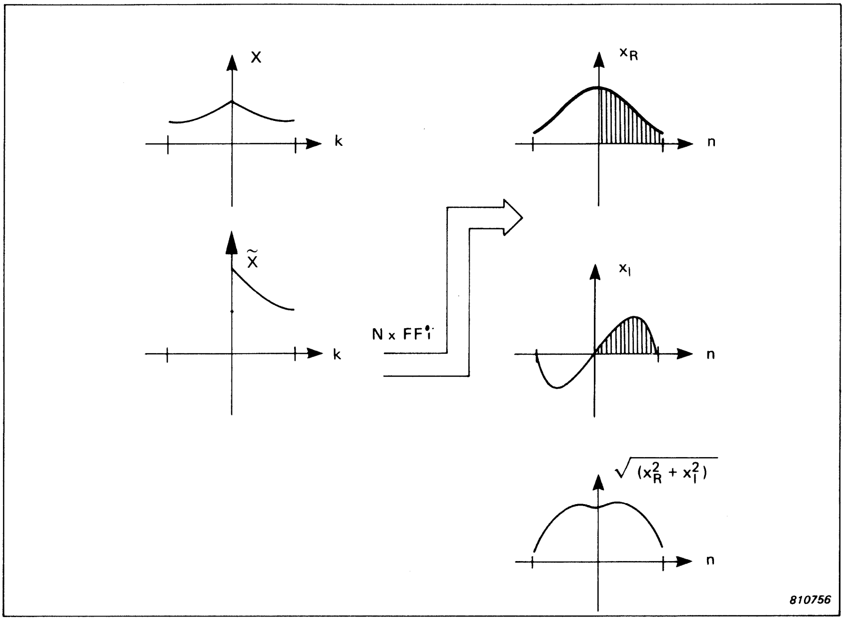


Fig. B2. Inverse transform of a real, even spectrum

B3. Conjugate Even Spectrum
Any complex spectrum can be inverse transformed by transforming the real and imaginary components separately by means of the procedure B1. However, this requires two Fourier transformations as well as some extra storage capacity for the intermediate results.

In the situation where the spectrum is conjugate even (i.e. corresponding to a real time signal) the following procedure can be used. This requires only one transformation and a minimum of storage space.

It will be seen that:

$$\mathscr{F}^{-1}\{X(k)\} = \mathscr{F}^{-1}\{X_R(k) + i\,X_I(k)\}$$

$$= N\,[\mathscr{F}\{X_R(k)\} - i\,\mathscr{F}\{X_I(k)\}]$$

$$= N\,[\,\xi_R(n) + \xi_I(n)\,] \qquad \text{(B5)}$$

Moreover:

$$\mathscr{F}\{X_R(k) + X_I(k)\} = \xi_R(n) + i\,\xi_I(n) \qquad \text{(B6)}$$

where:

$$\xi_R(n) = \mathscr{F}\{X_R(k)\}$$

$$i\,\xi_I(n) = \mathscr{F}\{X_I(k)\}$$

The calculation procedure is thus as follows:

(a) Add the real and imaginary parts for positive and negative frequencies. In practice this means adding the imaginary parts to the real parts (of the positive frequency spectrum) for the first half of the

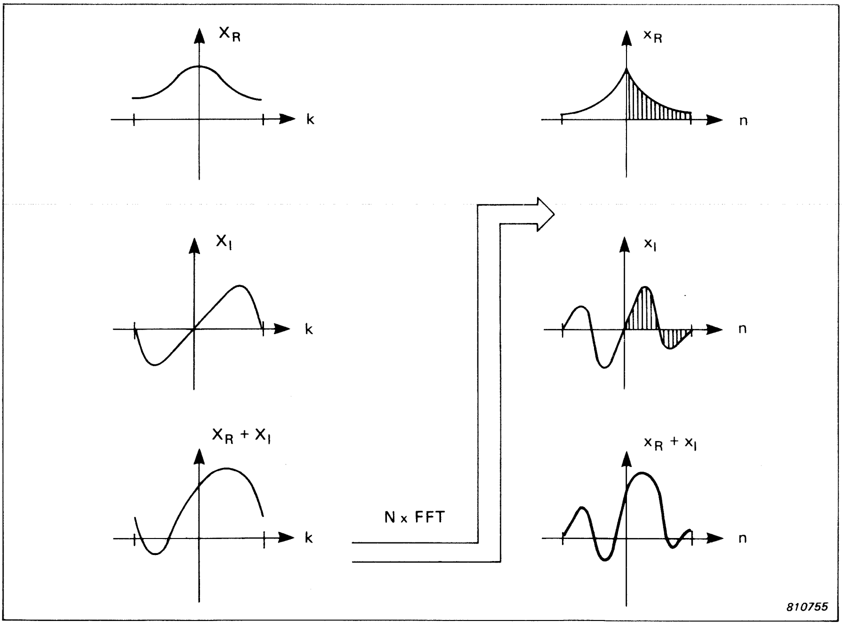


Fig. B3. Inverse transform of a conjugate even spectrum record and subtracting the same imaginary parts from the real parts for the second half (in reverse order).

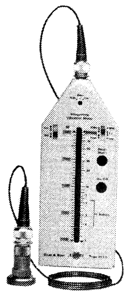
(b) Forward transform

(c) Add the real and imaginary parts for positive and negative time as in (a). Note that the negative time section will be located in the second half of the record and can be moved to its correct position before the first half. Zero time will then be in the centre of the record.

Fig.B3 illustrates this procedure.

News from the Factory

Integrating Vibration Meter Type 2513

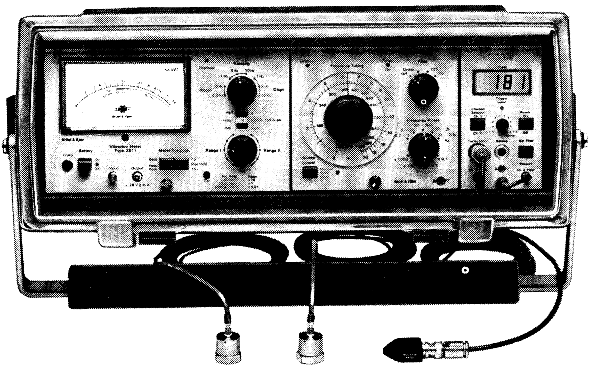


The new Integrating Vibration Meter Type 2513 brings a new concept in simplicity to precision vibration measurement. A compact, slim-line instrument intended mainly for on-site measurement, it features a 60 s L_{eq} facility for taking the ambiguity out of the measurement of fluctuating vibration sources. The integrator which gives the 2513 its name calculates a true mean-square average of the vibration over a period of one minute. It displays the 60 s L_{eq} on a thermometer-type LED display, with a constant 6% resolution over its 100-to-1 range. The display adjusts its brightness automatically to compensate for ambient light conditions, and can also be set to read current or maximum values of true-RMS or true-Peak levels.

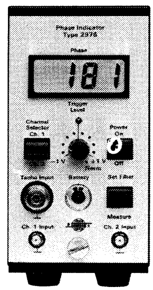
The 2513 can be set to measure either acceleration or velocity in the ranges 1 to 1000 m/s^2 and 0,1 to 100 mm/s respectively. It has a flat frequency response from 10 Hz to 10 kHz, and includes weighting filters for measuring Vibration Severity and Hand-Arm Vibration to ISO 2954 and ISO DIS 5349. The vibration transducer supplied is a robust Piezoelectric Accelerometer, Type 4384, which comes with a specially designed mounting magnet for ease of attachment to the vibrating object.

Principal applications of the new Vibration Meter will be machine-condition monitoring for maintenance scheduling. However it is also expected to find many uses in general vibration measurement, especially in the quality control of manufactured products (where its Severity setting will be of benefit), and the Hand-Arm weighting will enable it to be used for evaluating the human factors of portable power-tools.

Field Balancing Set Type 3517



Phase Indicator Type 2976



The Field Balancing Set Type 3517 combines in one robust case all the measuring instruments needed for balancing rotors in the field (without dismounting them from their own bearings) as well as for performing a wide range of vibration measurement and analysis tasks.

It can be used for both simple single plane balancing of, for example, flywheels and grinding wheels, and also for "Dynamic" two-plane balancing of rotating parts having distributed mass along a shaft. The method does away with strobe lights and the manual plotting of vector diagrams; the rather complicated two-plane balancing calculation is performed by a programmable pocket calculator. Ready programmed magnetic memory cards are available from B & K for the Texas Instrument SR 59 and the Hewlett-Packard HP 41, 67 and 97 calculators. The balancing set is equally suitable for multi-plane balancing where high resolution and accuracy is even more important, but larger calculators are needed.

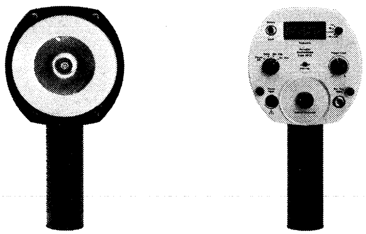
The Balancing Set, which is contained in a light-weight carrying-case of completely new design, consists of

a) General Purpose Vibration Meter Type 2511, which conditions the signal from the vibration pick-up, Accelerometer Type 4370, and displays the unbalance amplitude.

b) Tunable Filter Type 1621, which ensures that measurements are made at the rotational frequency only and suppresses other vibration components.

c) A Phase Indicator Type 2976, which displays unbalance phase with 1° resolution, and incorporates a built-in change-over switch for the two Piezoelectric Accelerometers Type 4370 included. It comes complete with an infra-red tachometer probe, and is available separately for users of the well-established Type 3513 Portable Vibration Analyzer to include field balancing in their activities. A special feature of the 2976 is its "Set Filter/Measure" switch, which enables the Filter to be tuned with great precision to the rotation frequency by reference to the phase shift of the tachometer signal when it is fed through the Filter.

Portable Stroboscope Type 4912



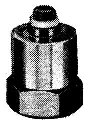
The portable Stroboscope Type 4912 is a compact hand-held instrument for qualitative investigation, and accurate measurement of various kinds of rapid repetitive motion in trouble shooting and in design and development situations.

The 4912 has three modes of operation. In the free running mode the stroboscope lamp flashes at the frequency set by the internal frequency generator and indicated on a 4 digit display to the nearest 0,1 Hz or r/min. The frequency range is from 5 to 125 Hz. In the "Tacho" mode the 4912 performs solely as a tachometer, displaying the frequency of the external trigger input signal in the range 0 — 1500 Hz or the equivalent r/min. In this mode the lamp does not light and the frequency display flashes when the upper frequency limit is exceeded. In the "Ext. Trigger" mode the 4912 can be used as a combined stroboscope and tachometer. The lamp flashes in synchronism with the external trigger input signal, the frequency of which is displayed. For frequencies in excess of 130 Hz the lamp will flash at the highest submultiple of the trigger frequency less than 130 Hz, and will continue to synchronize correctly with external trigger input frequencies up to 20 000 Hz. The display indicates frequencies up to 1500 Hz (90 000 r/min).

The 4912 can be triggered externally, via a coaxial cable, from a photoelectric tachometer probe (e.g. B & K MM 0012) a magnetic probe (e.g. B & K MM 0002), a vibration meter (e.g. B & K Type 2511), or a variety of other motion-sensing devices.

The stroboscope can be powered for 2,5 hours or more from rechargeable batteries built into its carrying handle or can be mains operated using Power Supply Type 2808. The integration of the light source, power supply and generator into a hand-held instrument weighing only 1,3 kg, makes the 4912 ideal for studies of moving machinery in cramped confined spaces and remote locations where transportability is essential.

Accelerometer Type 4384



Type 4384 is a uni-gain multi-purpose accelerometer, suitable for general shock and vibration measurements in industry, laboratory and education. It has a charge sensitivity of 1 pC/ms^{-2} (± 2 %), weighs 11 grammes and has a frequency range which is linear to within 10 % up to 12 kHz. The use of Delta shear® design gives it good all-round specifications, particularly low sensitivity to temperature transients (0,8 ms^{-2}/°C) and to base strains (0,02 ms^{-2}/$\mu\varepsilon$).